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The combined effects of delay and probability in discounting

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Abstract

Human discounting studies have frequently observed hyperbolic discounting of rewards that are delayed or probabilistic. However, no studies have systematically combined delay and probability in a single discounting procedure. Indifference points of hypothetical money rewards that are both delayed and probabilistic were determined. Probabilities were converted into comparable delays according to the *h*/*k* constant of proportionality determined by Rachlin et al. (1991), and discounting rates were calculated. These data provided a very good fit to the hyperbolic model of discounting, suggesting that delay and probability can be combined into a single metric in studies of discounting. The inclusion of a magnitude condition found the Magnitude Effect commonly found in studies of temporal discounting. A temporal resolution of uncertainty condition found no effect. The present paper offers a novel statistical method, within an established framework, for the analysis of data from studies of discounting that combine delay and probability.

Keywords: Delay discounting; Probability discounting; Self-control; Human

Experimental studies have often conceptualized impulsiveness and self-control as two ends in a continuum: Impulsiveness is exhibited in preference for small, immediate gains over large gains delayed in time, and self-control is exhibited in preference for large gains delayed in time over small, immediate gains (Ainslie, 1992; Rachlin, 1995). The conceptualization of the impulsiveness/self-control continuum in this way is considered a good experimental analog for a variety of human choice situations including whether or not to exercise, consume illegal substances, or pursue advanced education. Studies of delay discounting quantify this continuum by determining the rate at which subjective value of an outcome in the future decreases as delay to that outcome increases.

Economic analyses of delay discounting assume that the rate of discounting is exponential, where the reduction in value is proportional at each unit of time.

$$v_{\mathrm{d}} = Ve^{-kd} \tag{1}$$

In this equation, the discounted value of an outcome (v_{d}) is equal to the product of the undiscounted value (V) and a constant (e) raised to the inverse of the product of delay (d) and a discounting parameter (k). This free parameter (k) provides a measure of the degree to which the value of a reward is discounted when it is delayed, with higher values of k indicating greater discounting and more impulsiveness. However, if the value of k is assumed to be constant within subjects, the exponential model fails to account for the preference reversals commonly observed in studies of choice (see Green and Myerson, 1993 for relevant issues).

Preference reversal refers to the observation that preference for an immediate reward over a delayed reward tends to switch to the other alternative as an equal delay is added to both alternatives. A model of delay discounting, with a constant value of k within subjects, must predict such preference reversals. That is the case of hyperbolic functions such as Mazur's (1987) hyperbolic discounting model. This model not only accounts for preference reversals, but has been found to describe human and non-human animal behavior in inter-temporal choice more accurately than the exponential model (e.g. Kirby, 1997; Kirby and Markovic, 1995), accounting for a greater proportion of the variance.

$$v_{\mathrm{d}} = \frac{V}{1+kd} \tag{2}$$

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In this equation, the discounted value of an outcome (v_d) is equal to the ratio of the undiscounted value (*V*) and the quantity one plus delay (*d*) multiplied by a discounting parameter (*k*). Like the exponential model, higher values of *k* indicate more impulsiveness.

Mazur's (1987) discounting parameter has successfully differentiated, in the predicted direction, many populations thought to differ in impulsiveness. Delay discounting procedures have found that children are more impulsive than adults (Green et al., 1994, 1996), males are more impulsive than females (Kirby and Markovic, 1996), and pathological gamblers (Alessi and Petry, 2003; Petry, 2001) and various drug-dependent populations (Bickel and Marsch, 2001) are more impulsive than controls. Different delay discounting rates have been observed in clinical populations when DSM-IV diagnoses were used as predictors of impulsive behavior (Crean et al., 2000).

In addition to delay, the value of an outcome also decreases as a function of its probability of occurrence: $10 with a 0.5 probability is worth less than $10 with certainty for the vast majority of people. Some researchers have proposed that discounting of probabilistic rewards is a result of discounting due to expected waiting time to receive the reward (Rachlin et al., 1986, 1991), that a probabilistic event is one that occurs with a relative frequency over a series of repeated opportunities for its occurrence. For example, when a coin toss is repeated over a series of trials, saying that its probability of landing on heads is 0.5 means that heads will be the outcome in about one half of the trials (the larger the series of trials, the more the relative frequency of obtained heads will approximate the nominal probability). Rachlin et al. (1986) point out that, over these repeated trials the target event will sometimes occur after the first toss, other times after the second, the third and so on. This generates a geometric distribution of expected delays to the event, from which one can estimate the average expected time to its occurrence. Rachlin et al. argue that it is this average expected delay to the reward that leads to the discounting of its value. Low probabilities lead to long average expected delays and are in turn discounted more than high probabilities which lead to short average expected delays. Based on this interpretation, the study of the effect of probability is fundamentally the study of the effect of delay, and the basic process of discounting should conform to Mazur's (1987) hyperbolic discounting model, with odds-against winning (an estimation of the average expected waiting time) replacing delay in the equation.

$$v_d = \frac{V}{1 + h\theta} \quad (3)$$

The discounted value of an outcome (v_d) is equal to the ratio of the undiscounted value (*V*) and the quantity one plus odds-against ($\theta = (1 - \text{probability})/\text{probability}$) multiplied by a discounting parameter (*h*). Rate of probability discounting has been found to fit this hyperbolic function (Green et al., 1999; Rachlin et al., 1991).

In contrast to the position that the delay inherent in probabilistic events is responsible for probability discounting, others (e.g., Green and Myerson, 1996; Stevenson, 1986) have proposed that delay discounting is due to the uncertainty inherent in the delay to a reward; the degree of uncertainty about collecting a delayed reward increases as delay increases, and this increase in uncertainty results in discounting. Discounting due to probability (degree of uncertainty) is then more fundamental because uncertainty regarding the arrival of the reward is the cause of discounting. Based on this interpretation, the study of delay is fundamentally the study of probability.

Arguments for or against each of these positions aside, both conclude that the effect of delay and probability discounting on the value of rewards can be reduced to one common discounting process. The purpose of the present experiment is to examine the combined discounting effects of delay and probability. We know of no discounting studies that have determined indifference points for outcomes that are both delayed and probabilistic. Though some decision-making studies have included one alternative that was both delayed and probabilistic (e.g. Keren and Roelofsma, 1995; Navarick, 1987; Sagristano et al., 2002), these were not discounting studies and made no attempt to assign a value (indifference point) to the delayed and probabilistic alternative. Though the experiential discounting task (EDT; Reynolds, 2006; Reynolds and Schiffbauer, 2004) does determine indifference points for rewards that are both delayed and probabilistic, all rewards were available with a single probability and model-fitting analyses did not incorporate the specific probability. The present procedure is not only novel, but allows for a parametric examination of the combined effect of delay and probability as well as explore the possibility that results may be consistent with results from previous studies examining delay and probability discounting independently.

In the present experiment, indifference points were determined for hypothetical money rewards that were both delayed and probabilistic. Rachlin et al. (1991) previously determined subjectively equivalent delays and probabilities by asking participants to choose between hypothetical money that was delayed and hypothetical money that was probabilistic. For instance, participants were asked choose between $1000 delayed by 6 months and $1000 with various probabilities. Systematic variation of the probability allowed Rachlin et al. to determine the probability at which $1000 delayed by 6 months was subjectively equivalent. Delay and probability was equated in this manner, resulting in the constant of proportionality and allowing for delay and probability to be unified into a single metric.

$$d_{\text{equivalent}} = 35.3\theta \quad (4)$$

Eq. (4) converts odds-against (θ: determined from explicit probability) to an equivalent subjective delay ($d_{\text{equivalent}}$) that reduces reward by the comparable amount (or vice versa). This equivalent delay can then be added the explicit delay to determine a composite (Eq. (5)) that should result in discounting due to the explicit delay and probability.

$$\text{composite delay} = d + d_{\text{equivalent}} \quad (5)$$

When events are both delayed and probabilistic, the composite delay combines discounting from both delay and probability into a single metric (delay). For instance, a 25% probabilistic reward delayed by 1 week has a composite delay of 112.9 days. This composite delay is the sum of 105.9 days (θ of 25% probability = 3; from Eq. (4)) and 7 days (1-week delay).

$$v_{\mathrm{d}} = \frac{V}{1 + k(d + 35.3\theta)} \quad (6)$$

Eq. (6) is Mazur's (1987) hyperbolic equation when composite delay (calculated from Eqs. (4) and (5)) replaces the previous explicit delay. If discounting due to delay and probability can be equated in this manner, indifference points should decrease hyperbolically (rather than exponentially) regardless of the individual components that make up the composite delay; a high probability/long delay reward and a low probability/short delay reward may be equally discounted because both could yield similar composite delays. Alternatively, data more consistent with the exponential model would indicate that the subjective effect of delay and probability together are dissimilar to those of delay and probability independently (i.e. do not result in hyperbolic discounting).

Given the relevance of the magnitude variable in the analysis of discounting, \$10 and \$1000 magnitude conditions were included in the design. Furthermore, results from decision-making research show that the point at which the outcome of a probabilistic event is known relative to a delay (temporal resolution of uncertainty) is an important variable in choice (Arai, 1997; Sagristano et al., 2002; Wu, 1999). For this reason, we included early and late temporal resolution of uncertainty conditions (i.e. conditions where the outcome was known before the beginning of the delay to the reward or at the completion of that delay).

1. Method

1.1. Participants

Twenty-nine college students between 18 and 22 years of age were recruited for this study. These participants were recruited from a psychology departmental subject pool, and received credit in an introductory level class for participation. One participant was excluded from the analysis due to incomplete data. A second participant provided a natural logarithm-transformed probability discounting parameter greater than 3 standard deviations from the mean, and was excluded as an outlier. Twenty-seven participants were included in the analyses (seven males; mean age of remaining participants was 18.63).

1.2. Apparatus and instruments

A paper and pencil test of delay and probability discounting was employed to obtain indifference points in this choice procedure. There were four versions of the test: a delay discounting (DD) version, a probability discounting (PD) version, and two versions where probability and delay were combined and that differed as a function of temporal resolution of uncertainty: a Delay–Probability (DPD) and a Probability–delay discounting (PDD) versions.

1.2.1. Delay discounting procedure (DD)

Participants were asked to imagine possessing a winning lottery ticket. The prize could be collected after some delay or the ticket could be sold immediately to a lottery agent for a sum of money. Each page of the questionnaire had a unique combination of prize magnitude (\$10 or \$1000) and delay until prize availability (1 day, 1 week, 1 month, 6 months, 1 year, or 5 years). A column of money amounts in 2.5% increments of the prize value in either ascending or descending order (from top of page) was listed below the prize conditions. Participants were asked to mark an “X” next to each amount that they would be willing to accept from the lottery agent immediately to sacrifice their ticket. They were asked to assume that no judgments were being made about participation in a lottery and no laws were being broken.

1.2.2. Probability discounting procedure (PD)

Participants were asked to imagine possessing a lottery ticket with a known probability of winning a prize that could be collected immediately if a winner. However, it could be sold to a lottery agent before this determination. The lottery ticket had one of five probabilities (95%, 75%, 60%, 40%, 25%, or 5%) of winning the prize (\$10 or \$1000). Like the DD procedure, 2.5% increments of the prize value in either ascending or descending order were listed on each page.

1.2.3. Delay–probability discounting procedure (DPD)

Participants were asked to imagine possessing a lottery ticket with a known probability of winning a prize. This would only be determined following a delay. However, it could be sold to a lottery agent immediately for a sum of money. The same delays and probabilities from the DD and PD procedures were employed, with the exception of the 1-week delay and 40% probability (to reduce the duration of the procedure).

1.2.4. Probability–delay discounting procedure (PDD)

Participants were asked to imagine possessing a lottery ticket with a known probability of winning a prize. This would be determined immediately, but if the ticket was a winner, the prize could only be collected after some delay. However, it could be sold to a lottery agent immediately for a sum of money. The same delays and probabilities of the DPD condition were employed.

1.3. Procedure

In one 1.5-h experimental session, participants completed all procedures. Participants began the choice procedure after reading the directions and asking questions if necessary. The DD and PD conditions always preceded the DPD and PDD conditions. The order of DD and PD, and DPD and PDD were counterbalanced. Two magnitudes (\$10 and \$1000) as well as the order of money amounts (ascending and descending) were also counterbalanced. The different delays, probabilities, and delay/probability combinations within each block were random.

1.4. Data scoring and analysis

Indifference points were calculated as the arithmetic mean of the smallest value for which the participant would sell the lottery ticket in the ascending condition and the largest value for which the participant would not sell the lottery ticket in the descending condition. Nonlinear estimation using Statistica 5.0 was used to determine discounting parameters for the DD procedure according to Eqs. (1) and (2). To obtain discounting parameters from the PD procedure, probability of winning was converted to θ, and parameters were obtained according to Eq. (3). The 1-week delay and 40% probability were excluded in all analyses to maintain continuity with the DPD and PPD procedures. Indifference points are always reported as proportion of the standard amount.

Eq. (6) was used to obtain hyperbolic discounting parameters from the DPD and PDD conditions. Inferential analyses with discounting parameters required data transformation because the distributions of scores were positively skewed. Natural logarithm transformations were conducted; this normalized the distributions, and analyses were conducted with these transformed data.

2. Results

Individual indifference points from the DD condition were fit to the exponential and hyperbolic models of discounting (Eqs. (1) and (2), respectively). Goodness-of-fit measures (R^2) from these equations were compared with analysis of variance (ANOVA) to determine which model described the data more accurately. As expected, the hyperbolic model accounted for a greater proportion of the variance ($\bar{X}_{R^2} = 57\%$) than the exponential model ($\bar{X}_{R^2} = 52\%$, (1, 26) = 22.81, $p < 0.01$). A comparison of the magnitude condition was conducted with ANOVA using log-transformed discounting parameters obtained from the hyperbolic model. The Magnitude Effect was observed, with the smaller magnitude ($\bar{X}_{10} = -6.49$, S.E.M. = 0.38) discounted more than the larger magnitude ($\bar{X}_{1000} = -8.12$, S.E.M. = 0.31; $F(1, 26) = 22.17$, $p < 0.05$). A correlation of discounting parameters between $10 and $1000 magnitude conditions was positive and statistically significant ($r = 0.51$, $p < 0.05$). The plot of the hyperbolic model with median indifference points is shown in Fig. 1 (top).

Individual indifference points from the PD condition were fit to a hyperbolic model (Eq. (3)) and the exponential model similar to Eq. (1). As in the DD condition, the hyperbolic model accounted for a greater proportion of the variance ($\bar{X}_{R^2} = 78\%$) than the exponential model ($\bar{X}_{R^2} = 63\%$, $F(1, 26) = 81.13$, $p < 0.01$). An ANOVA was conducted with probability discounting parameters obtained from the hyperbolic model following log-transformations. The Reverse-Magnitude Effect was observed, with the smaller amount ($\bar{X}_{10} = -0.47$, S.E.M. = 0.16) discounted less than the larger amount ($\bar{X}_{1000} = -0.07$, S.E.M. = 0.16; $F(1, 26) = 4.36$, $p < 0.05$). A correlation of discounting parameters between $10 and $1000 magnitude conditions was positive and non-significant ($r = 0.28$, $p > 0.05$). The plot of the hyperbolic model with median indifference points in shown in Fig. 1 (bottom).

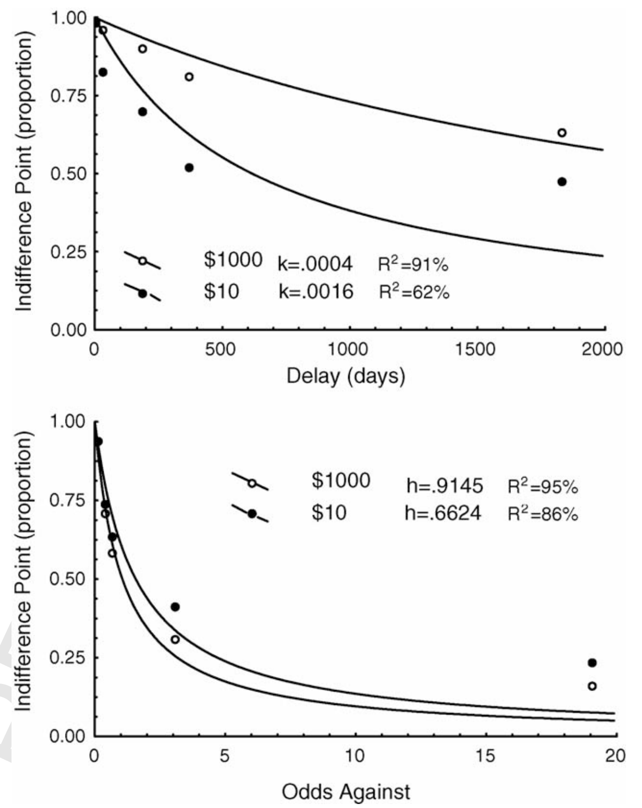

Fig. 1. Hyperbolic discounting curve obtained from median indifference points as a function of delay (top) and odds-against (bottom).

DPD and PDD conditions were compared to detect differences due to the temporal resolution of uncertainty condition. An ANOVA comparing indifference points from the DPD and PDD condition was not significant ($F(1, 26) = 0.03$, $p > 0.05$). Furthermore, interactions with Magnitude ($F(1, 26) = 1.55$, $p > 0.05$), Delay ($F(4, 104) = 0.87$, $p > 0.05$), and Probability ($F(4, 104) = 1.13$, $p > 0.05$) were not statistically significant. The DPD and PDD conditions were combined for all further analyses and are referred to as the combined condition from this point.

Individual data from the combined condition were fit to the hyperbolic (Eq. (6)) and the equivalent exponential models. The hyperbolic model's goodness-of-fit to individual data ($\bar{X}_{R^2} = 66\%$) was superior fit to the exponential model ($\bar{X}_{R^2} = 52\%$; $F(1, 26) = 53.71$, $p < 0.01$). ANOVA was conducted with log-transformed discounting parameters from the hyperbolic model. The Magnitude Effect consistent with delay discounting was observed, with the smaller amount ($\bar{X}_{10} = -6.82$, S.E.M. = 0.16) discounted more than the larger amount ($\bar{X}_{1000} = -7.48$, S.E.M. = 0.17; $F(1, 26) = 13.84$, $p < 0.05$). A correlation of discounting parameters between $10 and $1000 magnitude conditions was positive and significant ($r = 0.44$, $p < 0.05$). Median indifference points from the Combined condition were fit to Eq. (6). The plot of the model is shown in Fig. 2 (top). Indifference points from the participant with the best overall fit to Eq. (6) were plotted (Fig. 2, bottom).

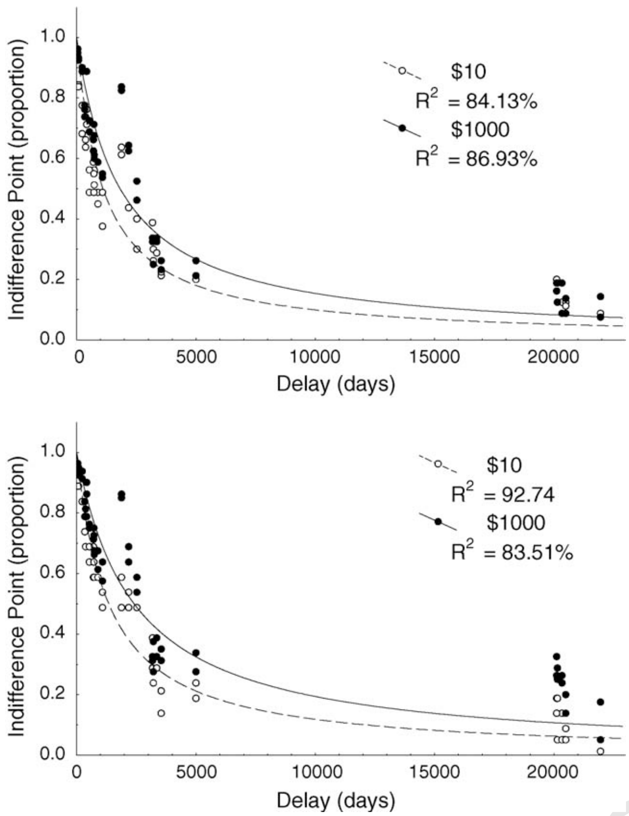

Fig. 2. Hyperbolic discounting curve obtained from median indifference points (top) and indifference points from individual providing the best-fit (bottom) as a function of composite delay.

3. Discussion

Results from previous research on delay discounting and probability discounting were replicated. The subjective value of delayed hypothetical rewards decreased as delay increased, and the rate of decline was consistent with Mazur's (1987) hyperbolic model of discounting. Furthermore, the Magnitude Effect was observed, with large rewards discounted less than small rewards. Subjective value of hypothetical probabilistic rewards decreased as odds-against increased (probability decreased), and the rate of decline was consistent with Rachlin et al.'s (1991) hyperbolic model of discounting. Furthermore, the Reverse-Magnitude Effect consistent with previous probability discounting research was observed.

Rachlin et al.'s (1991) constant of proportionality was used to determine the comparable delay to each of the probabilities used in the current procedure (using Eq. (4)). Once probabilities were converted to delays, the converted delay was added to the explicit delay to obtain a composite (Eq. (5)) for each delay/probability combination used in the present study. Indifference points as a function of these composite delays were fit to hyperbolic and exponential models of delay discounting. As in studies of delay discounting. Mazur's hyperbolic model of discounting provided a superior fit to the data (as compared to the exponential model). Most indifference points fall right along the best-fitting hyperbola in Fig. 2, and the goodness-of-fit measures are high. In fact, the R^2 from median indifference points obtained in the combined condition (84% and 86% for $10 and $1000 conditions, respectively) are equivalent to those obtained in the delay discounting procedure (62% and 91% for $10 and $1000 conditions, respectively), a procedure whose results are firmly established in the literature. Individual R^2 were in fact non-significantly higher in the Combined condition as compared to the DD condition ($F(1, 26) = 2.14$, $p > 0.05$). Certainly, the goodness-of-fit measures may have been somewhat inflated by the large number of indifference points in the combined condition. However, this inflation cannot be solely due to the presence of more indifference points; indifference points must conform to the hyperbolic discounting model to increase goodness-of-fit measures. The conformation of the data and composite delays to Mazur's (1987) hyperbolic model of discounting, simply by visual inspection of Fig. 2, is compelling. When delays were converted into comparable probabilities using the same constant of proportionality, the data and composite odds-against fit Rachlin et al.'s (1991) hyperbolic discounting model equally well. The composite delay proved to be a single metric that was useful across the range of delays and odds-against included as component parts. In this manner, some points in Fig. 2 consist of cases whose composite delay values are similar, but resulted from the combination of a low probability with a short delay and a high probability with a long delay. The fact that the fit of the hyperbolic model is good indicates that discounting was similar, independent of the specific probability and delay components. We are unaware of any other studies that have examined the combined discounting due to delay and probability in this manner.

Like previous studies of delay discounting, the Magnitude Effect was observed in the combined condition. This observation by itself is very interesting because the effect of magnitude in rewards that are both delayed and probabilistic have never been examined. Based on the contrasting observations in studies of delay and probability discounting, one might have expected no effect of magnitude when delay and probability were combined; if the Magnitude and Reverse-Magnitude Effects effectively neutralize each other, discounting rate of the $10 and $1000 rewards should have been similar. In contrast, the observed data indicate that the effect of delay predominates the effect of probability on magnitude. Though the present study offers little insight into the cause of this observation, the exploration of a possibility is relevant here.

Some decision-making research, while not parametrically combining delay and probability in a single alternative, have examined preference when at least one of the two alternatives was both delayed and probabilistic. Keren and Roelofsma (1995), for instance, offered one alternative that was immediate and certain versus another alternative that was both uncertain and delayed. While that study made no attempt to determine subjective value (as in studies of discounting), it did include numerous delays and probabilities. A strong preference for an immediate reward relative to a larger, delayed reward was initially observed. However, when both rewards were made equally unlikely ($p = 0.50$), the delayed reward was generally preferred. When an equal delay was added to both outcomes, individuals generally preferred the larger, more delayed reward, and this preference was not affected by probability of winning. These results indicate that probability has a strong influence in the near future (i.e. in the case of immediate outcomes), and this influence decreases as delay increases.

As applied to the present study, the results of Keren and Roelofsma (1995) indicate that the effect of probability on subjective value was reduced at longer delays. It is therefore possible that as delays increased, the influence of probability manifest in the Reverse-Magnitude Effect was diminished. Though the longest delay employed in the present experiment was only 5 years, any temporal distance from the immediate may be significant enough to affect college students in this manner. Additional research is necessary to comment further on the Magnitude Effect resulting from the combined effect of delay and probability discounting. Furthermore, this interaction of delay and probability indicate that a constant of proportionality used for the purpose of determining a composite delay should depend on the values of the component delays and probability. Nonetheless, any constant of proportionality is essentially an estimate for all individuals at all delays and probabilities. Given the high goodness-of-fit measures, Rachlin et al.'s (1991) constant of proportionality appears to be a very good estimate.

One weaknesses of the present study was the use of hypothetical rewards. Previous examination of possible differences in the delay discounting of real and hypothetical rewards have found no difference (Johnson and Bickel, 2002; Madden et al., 2003, 2004). We know of no discounting research that has compared real and hypothetical rewards that are probabilistic or delayed and probabilistic. Though a cover story was used to encourage behavior consistent with real rewards, there is no guarantee that participants' behavior reflected this.

Careful examination of Fig. 2 indicates some systematic deviation of indifference points from the best-fitting hyperbola. Specifically, some indifference points between 2000 and 4000 on the composite-delay axis are much higher than predicted by the hyperbola. These indifference points are from composite delays that include the 5-year explicit delay. We have no explanation for this aberrant finding other than the population from which we sampled (college students). Most students presumably expected to be out of college in 5 years, and such a significant and expected life-change might have had some unusual effect. We excluded indifference points obtained from composite delays that included the 5-year explicit delay and did not find a substantive change in discounting rate or goodness-of-fit.

Consideration of the 5-year delay brings up a second weakness of the present study: the difference in range of delays and probabilities. The range of probabilities employed in the present study was fairly large, resulting in median indifference points from 0.16 to 0.94 for the $1000 reward. However, the lowest median indifference point for a delayed $1000 was 0.64 (resulting from the 5-year delay). A more comprehensive procedure would have included a range of delays that led to a similar range of indifference points as that obtained from probability. Certainly, the inclusion of delays longer than 5 years (e.g. 25 years, 50 years) might have accomplished this. A quick review of delay discounting research employing longer delays (ignoring procedural differences) suggests however that median indifference points frequently do not fall below 0.25 even at a 25-year delay (e.g. Baker et al., 2003; Yi et al., 2005). Thus it is unclear if longer delays would have provided any more useful data.

A third weakness of the present paper is our use of a constant of proportionality obtained in a different study (Rachlin et al., 1991). In an ideal scenario, delays and probabilities would have been equated for each participant in the present study, allowing for the calculation of unique and possibly more accurate constants of proportionality for each individual. Unfortunately, we did not have such a procedure. Though it is possible to contrive subjective equivalents using interpolation, interpolation itself would require a priori selection of the appropriate model of discounting. Given that a primary analysis of the present study was the comparison of hyperbolic and exponential models of discounting, such interpolation would not have been justified. Though the participants were not the same, the procedure of Rachlin et al. and the present study were similar on many dimensions (e.g. college students, paper and pencil questionnaires, two-alternative choice situations, $1000 hypothetical reward magnitude), and does provide some basis for our use of Rachlin et al.'s constant of proportionality as an estimate (as stated previously). And given that the constant still resulted in composite delays that resulted in very good fits to the hyperbolic model, the estimate was more than adequate, at least for the procedural nuances in the present study.

The present study was a novel approach to the study of delay and probability discounting. Results indicate that delay and probability can be combined into a single metric, and examination of indifference points as a function of this metric is consistent with the hyperbolic model of discounting. Preference for early temporal resolution of uncertainty was not observed with the present procedure. The Magnitude Effect was observed, consistent with the effects of delay. Additional research may provide answers to why this occurred as well as to the contrasting effects of magnitude on delay and probability.

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