Test Statistic and Confidence Interval Formulas in the Small Sample Case

One Population Mean (μ)
($n < 30$)

Test Statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$100(1-\alpha)\%$ Confidence Interval:

$$\bar{x} \pm t_{df, \frac{\alpha}{2}} \frac{s}{\sqrt{n}} \quad , df = n-1$$

where the assumption is that the sample is randomly selected from a normally distributed population

Difference in 2 Population Means ($\mu_1 - \mu_2$)
($n_1 < 30$ or $n_2 < 30$)

Test Statistic: $\dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ (where D_0 is usually 0)

where $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

(assuming the 2 population variances are equal)

(Independent Samples)

(The samples are randomly selected from normally distributed populations.)

$100(1-\alpha)\%$ Confidence Interval:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{df, \frac{\alpha}{2}} \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \quad , df = n_1 + n_2 - 2$$

Applies here also

Difference in 2 Population Means ($\mu_1 - \mu_2 = \mu_d$) (Matched Pairs Situation)

Test Statistic:

$$t = \frac{\bar{d} - \mu_{d_0}}{s_d/\sqrt{n_d}}$$ (where μ_{d_0} is usually 0)

$100(1-\alpha)\%$ Confidence Interval:

$$\bar{d} \pm t_{df, \frac{\alpha}{2}} \left(\frac{s_d}{\sqrt{n_d}}\right), df = n_d - 1$$

Assume n differences represent a random sample from a normal population.

Test Statistic and Confidence Interval Formulas in the Large Sample Case

One Population Mean (μ)

($n \geq 30$)

Test Statistic: $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$100(1-\alpha)\%$ Confidence Interval:

$$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

One Population Proportion (p)

Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$

where $np_0 > 5$ and $nq_0 > 5$

$100(1-\alpha)\%$ Confidence Interval:

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

where $n\hat{p} > 5$ and $n\hat{q} > 5$

Difference in 2 Population Means ($\mu_1 - \mu_2$)

($n_1 \geq 30$ and $n_2 \geq 30$)

Test Statistic: $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ (where D_0 is usually 0)

$100(1-\alpha)\%$ Confidence Interval:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Difference in 2 Population Proportions ($p_1 - p_2$)

Test Statistic:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}}$$

where $n_1\hat{p}_1 > 5$ and $n_1\hat{q}_1 > 5$ and $n_2\hat{p}_2 > 5$ and $n_2\hat{q}_2 > 5$

$100(1-\alpha)\%$ Confidence Interval:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

where $n_1\hat{p}_1 > 5$ and $n_1\hat{q}_1 > 5$ and $n_2\hat{p}_2 > 5$ and $n_2\hat{q}_2 > 5$